STATEMENT REPORT

under the procedure for acquisition of the educational and scientific degree "Doctor" by **Deyan Zhivkov Dzhundrekov** of the PhD Thesis entitled: "Graded Algebras and Noncommutative Invariant Theory", In the Scientific field: 4. Natural Sciences, Mathematics and Informatics Professional field: 4.5. Mathematics Doctoral program "Algebra, **Number Theory** and Applications - **Topology**", Department of Algebra, Faculty of Mathematics and Informatics (FMI), Sofia University "St. Kl. Ohridski" (SU),

The statement report has been prepared by: Prof. Dr. Jörg Koppitz, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

as a member of the scientific jury for the defense of this PhD thesis according to Order № **РД 38-64/1.2.2024** of the Rector of the Sofia University.

1. General characteristics of the dissertation thesis and the presented materials

The thesis consists of 71 pages and 13 additional pages conclusions, containing a list of nine items of main contributions, a list of two publications related to the thesis, a list of seven talks, in which scientific parts of the thesis were presented, a declaration of the originality, and an acknowledgment. Moreover, at the end of the paper there is an index (two pages) and a list of 56 publications, which were used in that thesis. The paper starts with the Context. The first chapter is the Introduction (five and a half pages). After the Introduction follow three mathematical chapters. Chapter 2 provides the preliminaries and is divided in five subchapters. It is the largest chapter (with 38 pages) of the thesis. Chapter 3 is divided in two subchapters containing six proved lemmas and four proved theorems. Finally, Chapter 4 contains three proved lemmas and two proved theorems.

2. Short CV and personal impressions of the candidate

Deyan Zhivkov Dzhundrekov was born in Bulgaria and is an Assistant Professor at the Sofia University at the Faculty of Mathematics and Informatics since 2020. In particular, D. Dzhundrekov was student of the Faculty of Mathematics and Informatics (SU) from 2013 until 2019, was graduated 2017 (Bachelor) and 2019 (Magister). He is a native speaker Bulgarian and speaks, writes, and reads English excellent. I met Dzhundrekov first at a scientific conference in February 2024. At this conference, he gave a presentation presenting the results of his PhD thesis. Unfortunately, I had not yet any personal contact with Dzhundrekov.

3. Content analysis of the scientific and applied achievements of the candidate, contained in the presented PhD thesis and the publications to it, included in the procedure

Invariant theory is a branch of abstract algebra. It is related to works by Lagrange and Gauß. The real invariant theory began with the works by Boole and Hesse. The problem of finite generation is a central one for invariant theory. Another important combinatorial question of invariant theory is, how many invariants there are, which was answered in 1897. The invariant theory was not more popular. In 1989, there appeared new questions and problems in

invariant theory. The noncommutative invariant theory began with a paper by Margarete Wolf in 1936. Now, it becomes interesting what classical invariant theory results can be generalized to the noncommutative case. In fact, there are not many such classical results that could be transformed. So, noncommutative invariant theory was dead also for a while. The first interesting results in noncommutative invariant theory were given in a Russian paper by A. N. Koryukin in 1984. The author considers reductive groups. His idea was to define a such called S-action, which allowed to simulate commutativity. This idea was leading for the thesis by Deyan Dzhundrekov. It appears the following fundamental problem of invariant theory. Let G be a reductive group and find a fundamental system of generators of the algebra of the (noncommutative) polynomials, invariant under the action of G. The presented thesis provides a complete answer for the case when G is a symmetric group (Chapter 3) and when G is the alternative group for polynomials in d noncommuting variables (Chapter 4). So the thesis provides an essential contribution to the noncommutative invariant theory since it gives the answer of an important problem in that theory for an essential class of groups.

4. Approbation of the result

a) The scientific works meet the national requirements (under Art. 2b, para. 2 and 3 of ADASRB) and, respectively, to the additional requirements of Sofia University "St. Kliment Ohridski" for acquiring the educational and scientific degree "Doctor".

b) The results presented by the candidate in the dissertation work and scientific works do not repeat such from previous procedures for acquiring a scientific title and academic position.

c) There is no plagiarism proven in the legally established order in the submitted dissertation work and scientific papers under this procedure. The candidate has two publications, both together with S. Boumova, V. Drensky, and M. Kassabov. One of the publications (in 2022) is in the journal "Türkish Journal of Mathematics" published by the "Scientific and Technological Research Council Turkey", which is "Q2" in 2022. The second publication is in the journal "Mathematics". "Mathematics" is an open access journal published by MDPI (Basel). I would evaluate the publication in the "Turkish Journal of Mathematics" as the more important one. The candidate has no further publications. Deyan Dzhundrekov was a member of a scientific project "Algebra and Combinatorial Structures". Deyan Dzhundrekov has given presentations at seven conferences, which is, in my opinion, an excellent number for a PhD student and it justifies the impact of the research of the candidate.

5. Qualities of the abstract

The abstract consists of 36 pages plus the complete list of references, and plus Conclusions. The abstract is almost the PhD thesis without proofs of the lemmas and theorems. The Introduction is one-to-one the Introduction of the PhD thesis. In my opinion, the Abstract should be more compact and not so similar to the PhD thesis. In particular, the second chapter could be reduced extremely since this Chapter does not contain essential own results.

6. Critical notes and recommendations

In my opinion, the relation between the length of the fundamental part (44 pages) and the main part of Dzhundrekov`s own results (26 pages) should be more devoted

to the main part. The thesis has an excellent list of references. The results are important enough to be published in another journal than in the journal "Mathematics" by the controversial publisher MPDI.

7. Conclusion

Having become acquainted with the PhD thesis presented in the procedure and the accompanying scientific papers and on the basis of the analysis of their importance and the scientific and applied contributions contained therein, I confirm that the presented PhD thesis and the scientific publications to it, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the Act on Development of the Academic Staff in the Republic of Bulgaria, the Rules for its Implementation and the corresponding Rules at the Sofia University "St. Kliment Ohridski" (FMI-SU) for acquisition by the candidate of educational and scientific degree "Doctor" in the Scientific field "Natural Sciences, Mathematics and Informatics", Professional field "Mathematics". In particular, the candidate meets the minimal national requirements in the professional field and no plagiarism has been detected in the scientific papers submitted for the competition.

Based on the above, I strongly recommend the scientific jury to award Deyan Zhivkov Dzhundrekov, the educational and scientific degree "Doctor in the Scientific field Mathematics and Informatics, Professional field Mathematics, Doctoral program "Algebra, Number Theory and Applications - Topology".

Date : 12.03.2024   Signature : Jörg Koppitz, Prof., Dr.